### Part 1: Constructing the 99% Confidence Interval

To construct a 99% confidence interval for the proportion of vaccine-eligible people who received the flu vaccine, we follow these steps:

1. \*\*Calculate the sample proportion (\(\hat{p}\)):\*\*

\[

\hat{p} = \frac{978}{2350} = 0.4162

\]

2. \*\*Determine the standard error (SE) of the proportion:\*\*

\[

SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{0.4162 \times (1 - 0.4162)}{2350}} = \sqrt{\frac{0.4162 \times 0.5838}{2350}} = \sqrt{\frac{0.2431}{2350}} = \sqrt{0.0001034} = 0.01017

\]

3. \*\*Find the critical value for a 99% confidence interval:\*\*

For a 99% confidence interval, we use the z-value corresponding to the 99.5th percentile of the standard normal distribution (since it's two-tailed). This value is approximately \( z = 2.576 \).

4. \*\*Calculate the margin of error (ME):\*\*

\[

ME = z \times SE = 2.576 \times 0.01017 = 0.0261

\]

5. \*\*Construct the confidence interval:\*\*

\[

\text{Lower Bound} = \hat{p} - ME = 0.4162 - 0.0261 = 0.3901

\]

\[

\text{Upper Bound} = \hat{p} + ME = 0.4162 + 0.0261 = 0.4423

\]

Therefore, the 99% confidence interval is:

\[

(0.3901, 0.4423)

\]

6. \*\*Comment on the belief:\*\*

The belief that 45% of vaccine-eligible people received the flu vaccine corresponds to \( p = 0.45 \). Since 0.45 is not within our confidence interval (0.3901, 0.4423), we can conclude that there is significant evidence to suggest that the true proportion of vaccine-eligible people who received the flu vaccine is less than 45% at the 99% confidence level.

### Part 2: Determining the Smallest Sample Size for Canada

To find the smallest sample size \( n \) that guarantees a margin of error of 0.02 or less for a 99% confidence interval, we use the formula for the margin of error:

\[

ME = z \sqrt{\frac{p(1 - p)}{n}}

\]

We want \( ME \leq 0.02 \). Since we don't have a prior estimate for \( p \) in Canada, we use the conservative estimate \( p = 0.5 \) which maximizes the product \( p(1 - p) \):

\[

0.02 = 2.576 \sqrt{\frac{0.5 \times 0.5}{n}}

\]

Solving for \( n \):

\[

0.02 = 2.576 \sqrt{\frac{0.25}{n}}

\]

\[

0.02 = 2.576 \times \frac{0.5}{\sqrt{n}}

\]

\[

0.02 = \frac{1.288}{\sqrt{n}}

\]

\[

0.02 \sqrt{n} = 1.288

\]

\[

\sqrt{n} = \frac{1.288}{0.02}

\]

\[

\sqrt{n} = 64.4

\]

\[

n = (64.4)^2

\]

\[

n = 4147.36

\]

Since the sample size must be an integer, we round up to the next whole number:

\[

n = 4148

\]

Thus, the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 is \*\*4148\*\*.